GEOMETRY

Seeing, Doing, Understanding

Third Edition

Harold R. Jacobs

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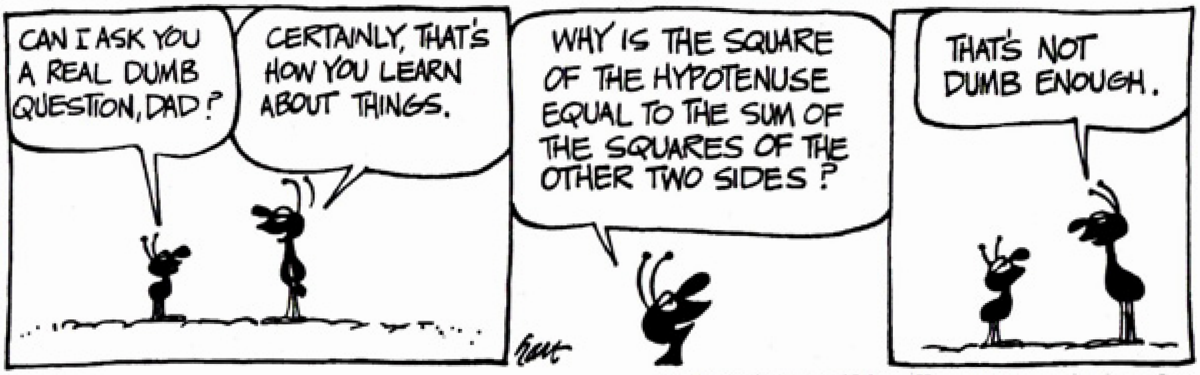

CAN I ASK YOU A REAL DUMB QUESTION, DAD?
CERTAINLY, THAT'S HOW YOU LEARN ABOUT THINGS.
WHY IS THE SQUARE OF THE HYPOTENUSE EQUAL TO THE SUM OF THE SQUARES OF THE OTHER TWO SIDES?
THAT'S NOT DUMB ENOUGH.
hart

GEOMETRY

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Third Edition

Harold R. Jacobs

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Forewords

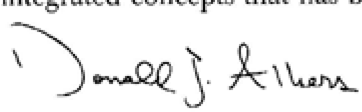
This is one of the great geometry books of all time. It is the book that I would like to have had for the geometry classes that I taught at Menlo School and College. It is the finest example of instructional artistry that I have encountered.

When I taught statistics courses, I used the counting and probability chapters from *Mathematics: A Human Endeavor* by Jacobs. His approach engaged my students and enabled them to discover and master this subject. In the present book, Jacobs does the same thing for geometry.

Harold Jacobs–author and master teacher–loves geometry, and this book shows it. Its thousands of photographs, diagrams, and figures draw the reader into the material. Jacobs has a genius for finding illustrations that reveal a subject and make it come alive. A look at his sources shows the broad search that he made to assemble this visual feast.

Teachers know that in most classes the goal for students is to complete the homework exercises with as little pain as possible. When students get their assignment, they first go to the exercises. If the teacher worked an example in class, the student follows that. If not, then the student scans the text in search of an example that is similar to the exercise. So many textbooks are little more than a bunch of examples, followed by exercises that can be solved by using the methods of the examples. And the student learns by creative copying.

As you page through this book, you will be struck by how few words Jacobs uses in introducing each lesson. Jacobs knows that students will probably not read much introductory material and want to get to the exercises as quickly as possible–solutions are still the goal. But, for Jacobs, the exercises are the beating heart of the book. You will first notice that they are beautifully illustrated with real-world material. As you read, you will discover that they are engaging, carefully sequenced, and structured so that students discover ideas for themselves. Students come away from his exercise sets empowered, because the ideas have become "their" ideas. Students using this book, working alone or in groups, will learn geometry by doing it. The result is that they will enjoy learning a body of integrated concepts that has become their intellectual possession.



Donald J. Albers
The Mathematical Association of America

Twenty-six hundred years ago Thales of Miletus launched the idea that Nature itself is subject to the laws of logic. Geometry, which had previously existed as a practical science, was found during the next few centuries to be tightly bound by logic. Some complex geometric facts were shown to be consequences of far simpler facts, and soon similar analyses of more general ideas about the world became standard for the Greek philosophers. The principle of stitching a few fundamental ideas together by logical analysis is at the heart of all modern science.

About 300 B.C.E., Euclid compiled his famous *Elements,* containing much of the geometry known at the time. Although the *Elements* does not fully meet today's standards of rigor, it quite rightly became and remained the principal textbook for teaching geometry for more than 2,000 years. In the past 100 years, however, the idea of *proof* has slowly receded (from American textbooks, at least), and now there is often little distinction between statements that have been proved formally and those that appear true on the basis of a drawing or a few measurements. This is an immense loss for American education, especially for science education. Some have tried to introduce formal proofs in algebra in place of those in geometry, but the results in algebra do not have the appeal of theorems such as that of Pythagoras. Geometry offers surprising theorems that can be proved by ingenious reasoning.

This book restores the idea of *proof* to its rightful place in geometry. Whoever studies this text will know what a proof is, what has been proved, and what has not.

Andrew M Gleason

Andrew M. Gleason
Harvard University

Photograph by Roy Bishop

A Letter to the Student

The most useful question in the history of science is, Why? Because mathematics is the language of science, the same question is important in mathematics as well.

The oldest standing examples of applied geometry are the pyramids of Egypt, built about 2600 B.C. The bases of these structures are almost perfect squares, and their faces are precisely constructed triangles. There is evidence that the Egyptians even knew how to compute the *volume* of a pyramid but no evidence that they ever tried to understand or explain their methods by asking, "Why?" Their mathematics was based on intuition and experience; even for other ancient civilizations from Babylon to Rome, intuition and experience were as far as geometry went. The subject consisted of disconnected rules, and the fact that these rules "worked" in measuring land and constructing buildings seemed to be enough.

Your understanding of geometry has been like that of these people of long ago. You have a good intuitive knowledge of the subject simply by growing up in the world. As a small child, you may have played with blocks in the shape of *cubes.* On the playground, you have played with balls in the shape of *spheres.* In school, you use paper ruled with *parallel* and *perpendicular lines.* You drink from cans in the shape of *cylinders* and eat ice cream from containers in the shape of *cones.* Geometry is everywhere, and so it is natural for you to take it for granted without ever asking, "Why?"

About 600 B.C., the Greeks made a discovery that led to a new way of looking at mathematics. Thales, one of the seven wise men of antiquity, saw how geometric ideas that had been discovered intuitively could be related to one another logically. If some were true, then others must follow without question. He raised the question Why? about a mathematical idea and then successfully answered it. For example, the four edges of the Great Pyramid that meet at its top are equal in length. Thales explained why it follows from this fact that the angles that these edges form with the sides of the base also must be equal.

After Thales came other Greek mathematicians, including Pythagoras, Hippocrates, and, eventually, Euclid. Euclid, about whom you will learn more in this book, presented geometry as a logical system of connected ideas so well that his book titled the *Elements* became the most widely used textbook ever written. Euclid's approach to geometry, centered on the question *why,* was so successful that it even inspired writers in other fields to organize their ideas in the same way. One of the most influential books of modern science, the *Principia* by Sir Isaac Newton, was modeled after Euclid's *Elements.*

The word *geometry* derives from the Greek words for "earth" and "measure" and in some languages is still used today to mean "earth measurement" or "surveying." Remarkably, although Euclid's *Elements* is the most famous geometry book of all time, the Greek word for geometry never appears in it. The *Elements* was not a book about measuring the earth; rather, by bringing geometry together as a coherent whole, it made geometry understandable *and* provided a model for other sciences.

As you pursue your study of geometry, you will encounter not only many familiar ideas but also many ideas new to you that are both beautiful and surprising. As the title of this book suggests, it is about seeing, doing, and understanding. As you read the lessons, you will *see* the main ideas and some of their applications. With the exercises comes your chance to *do, understand,* and, consequently, know *why.* I hope that in using this book you will discover for yourself the wonder and excitement of geometry so that you will find your study of it to be an enjoyable and rewarding endeavor.

Harold Jacobs

Acknowledgments

Since beginning work on the first edition of *Geometry* some 30 years ago, I have benefited from the comments and suggestions of many people. Many teachers have been generous in sharing their ideas, both at conferences and through correspondence, and students have contributed as well.

I am especially grateful to Peter Renz, for his unflagging effort and support during the development of all three editions of this book; to Donald J. Albers, Andrew M. Gleason, Keith Henderson, and Thomas Rike, for their detailed comments on the preliminary manuscript of the third edition; and to Patricia Zimmerman, for her expertise and patience in editing this edition and the first. I am also indebted to many colleagues throughout the United States for their various contributions and helpful advice: to Thomas Banchoff, James A. Barys, Charles Bigelow, Richard D. Bourgin, William R. Chambers, Charles Herbert Clemens, Kevin DeVizia, Russel L. Drylie, Kate Epstein, Dani Falcioni, Frederick P. Greenleaf, Shirley S. Holm, Kris Holmes, Jane Jelinek, Ellen Kaplan, Laurence Kaplan, Robert Kaplan, Lehman Kapp, John Larsen, Joseph Malkevitch, Otis W. Milton III, Harry D. Peterson, Gwen Roberts, Doris Schattschneider, Dennis Simons, Wolfe Snow, John W. Sperry, and Clem Wings, in regard to this edition, and to Dennis Anderson, Steven Bergen, Richard Brady, Don Chakerian, Barbara Fracassa, Gary Froelich, Hector Hirigoyen, Geoffrey Hirsch, Mel Noble, Linda Rasmussen, Sy Schuster, Bill Shutters, Ross Taylor, and Sharon Tello, in regard to earlier editions.

Putting a book such as this one together is a formidable task. Without the patience and support of the dedicated staff at W. H. Freeman and Company, it would not have been possible. For their valued efforts, many times above and beyond the call of duty, I would like to thank Mary Johenk, Craig Bleyer, Jane O'Neill, Paul Rohloff, Diana Blume, and Vikii Wong. I remain grateful to Robert Ishi and Heather Wiley for their efforts on earlier editions and their influence on the present one.

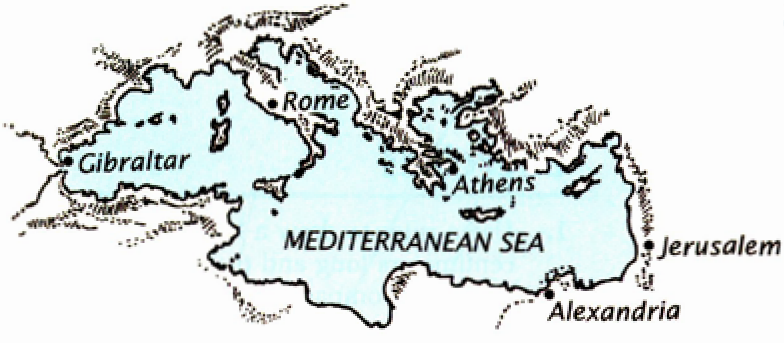
I would also like to acknowledge my indebtedness to other authors whose work has been especially helpful to me in this revision: Benno Artmann, H. S. M. Coxeter, John L. Heilbron, David W. Henderson, Dan Pedoe, and David Wells. Finally, I want to express my gratitude to Martin Gardner, for his seemingly endless ideas and inspiring enthusiasm, and to Howard Eves, Morris Kline, George Polya, and W. W. Sawyer, for showing us by example what good mathematics teaching is all about.

Old Euclid drew a circle
On a sand-beach long ago.
He bounded and enclosed it
With angles thus and so.
His set of solemn graybeards
Nodded and argued much
Of arc and of circumference,
Diameters and such.
A silent child stood by them
From morning until noon
Because they drew such charming
Round pictures of the moon.

VACHEL LINDSAY

INTRODUCTION

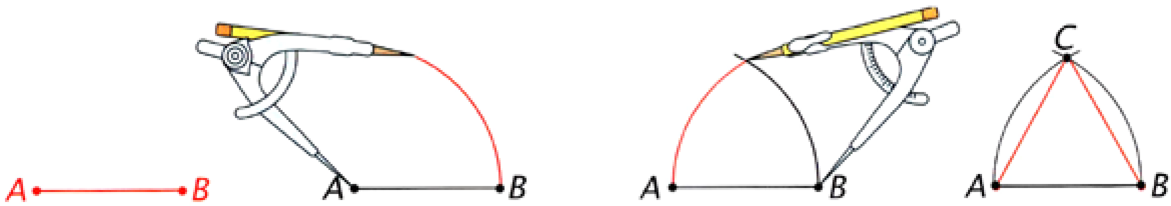
Euclid, the Surfer, and the Spotter



In about 300 B.C., a man named Euclid wrote what has become one of the most successful books of all time. Euclid taught at the university at Alexandria, the main seaport of Egypt, and his book, the *Elements,* contained much of the mathematics then known. Its fame was almost immediate. The *Elements* has been translated into more languages and published in more editions than any other book except the Bible.

Although very little of the mathematics in the *Elements* was original, what made the book unique was its logical organization of the subject, beginning with a few simple principles and deriving everything else from them.

The *Elements* begins with an explanation of how to draw an ***equilateral triangle***, that is, a triangle with three sides of equal length. Euclid's method requires the use of two tools: a straightedge for drawing straight lines and a compass for drawing circles. These tools have been used to make geometric drawings called ***constructions*** ever since Euclid's time.



To construct an equilateral triangle, we begin by using the straightedge to draw a segment for one side. The segment is named AB in the figure at the left above. We set the metal point of the compass at A and adjust the compass so that the pencil point falls on B. The compass is now set to draw a circle with radius AB about point A. We draw an arc of this circle above the segment AB as shown in the second figure. Next we move the metal point of the compass to point B and, keeping the compass set at the same radius, cross the arc that we just drew with a second arc, as shown in the third figure. Finally, we draw two line segments from the point of intersection (labeled C in the last figure) to points A and B to form the triangle.

Exercises

1. Use a ruler to draw a line segment 12 centimeters long and then use a straightedge and compass to construct an equilateral triangle having this segment as one of its sides.

Next, we will consider two geometric problems about equilateral triangles. To make them easier to understand, they will be presented in the form of a story.

The Puzzles of the Surfer and the Spotter

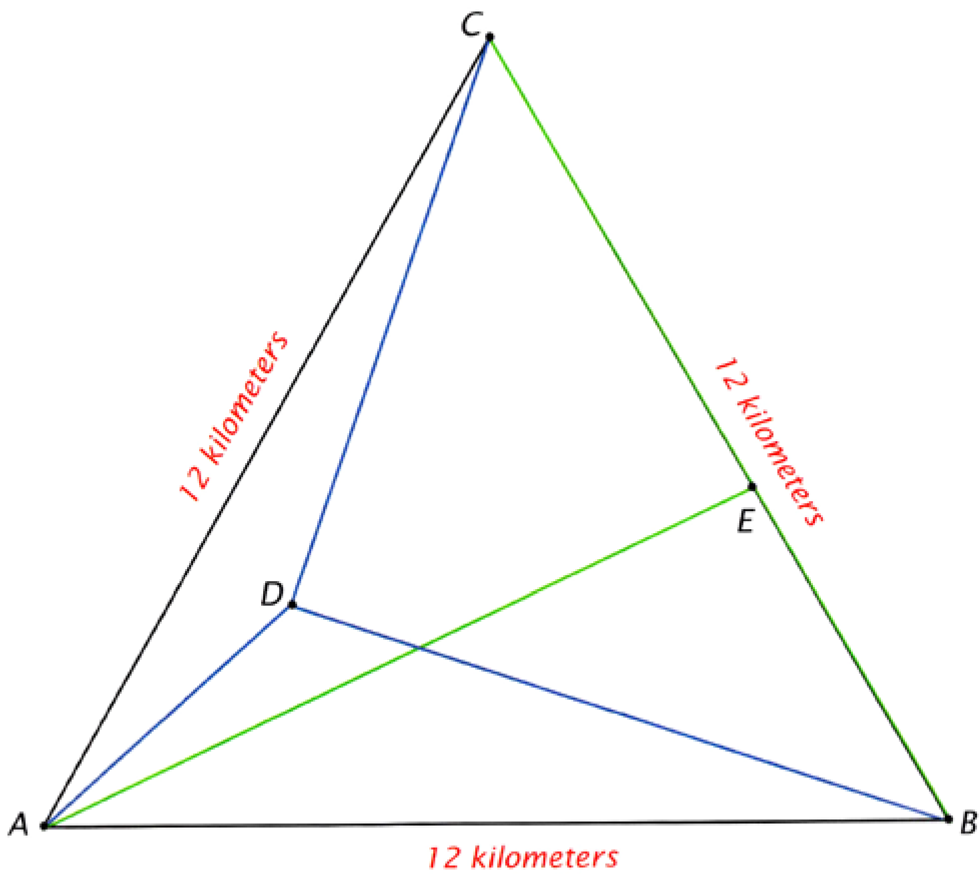
One night a ship is wrecked in a storm at sea and only two members of the crew survive. They manage to swim to a deserted tropical island where they fall asleep exhausted. After exploring the island the next morning, one of the men decides that he would like to spend some time there surfing from the beaches. The other man, however, wants to escape and decides to use his time looking for a ship that might rescue him.

The island is overgrown with vegetation and happens to be in the shape of an equilateral triangle, each side being 12 kilometers (about 7.5 miles) long.

Wanting to be in the best possible position to spot any ship that might sail by, the man who hopes to escape (we will call him the "spotter") goes to one of the corners of the island. Because he doesn't know which corner is best, he decides to rotate from one to another, spending a day on each. He wants to build a shelter somewhere on the island and a path from it to each corner so that the sum of the lengths of the three paths is a minimum. (Digging up the vegetation to clear the paths is not an easy job.) Where should the spotter build his house?

The figure below is a scale drawing of the island in which 1 centimeter (cm) represents 1 kilometer (km). Suppose that the spotter builds his house at point D. The three paths that he has to clear have the following lengths: DA = 4.4 km, DB = 9.1 km, and DC = 7.9 km. Check these measurements with your ruler, remembering that 1 cm represents 1 km. The sum of these lengths is 21.4 km.

If the spotter builds his house at point E, the path lengths are: EA = 10.4 km, EB = 5.1 km, and EC = 6.9 km, and their sum is 22.4 km. So point D is a better place for him to build than point E. But where is the best place?



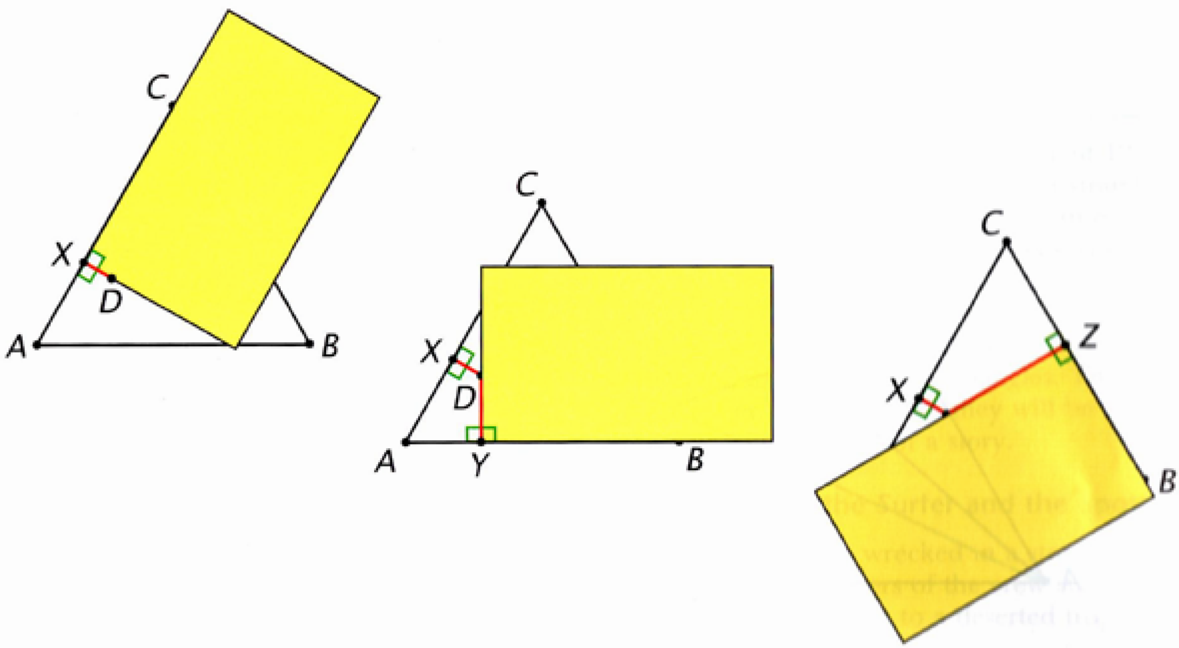
2. Use the equilateral triangle that you drew in the first exercise to represent the island. Choose several different points on it; for each point, measure the distance between it and each of the corners to the nearest 0.1 cm, and find their sum, as illustrated for points D and E on page 3.

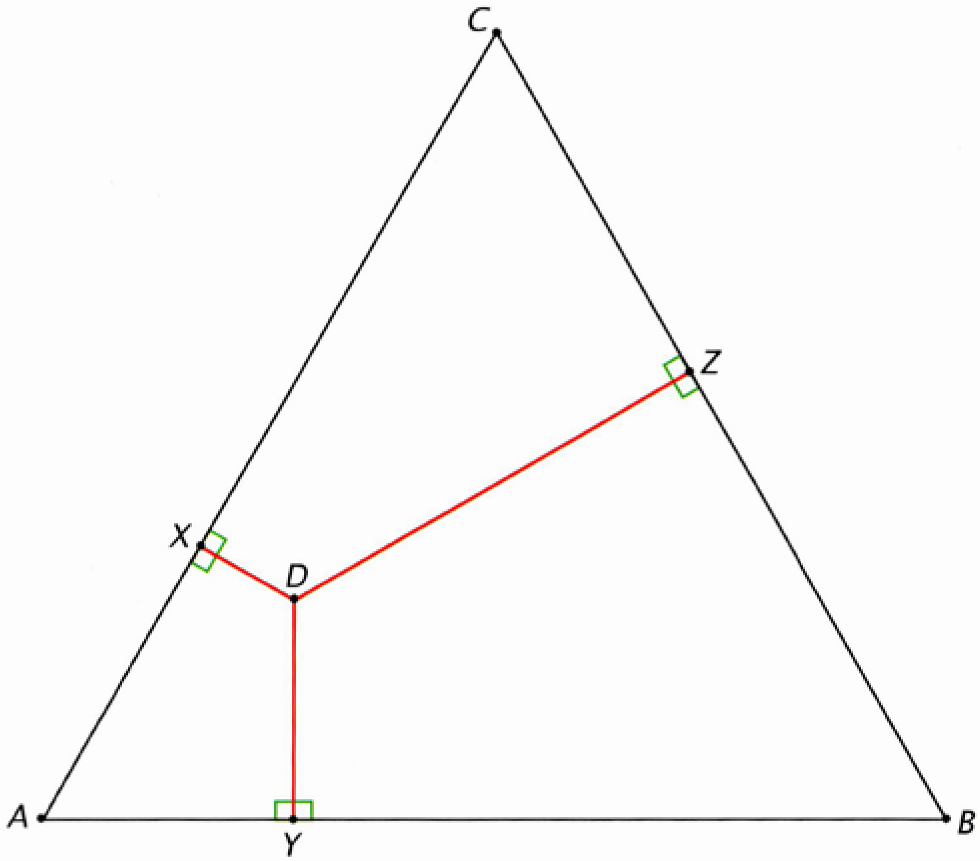
3. On the basis of your work in exercise 2, where do you think is the best place for the spotter to build his house? And how many kilometers of path does he have to clear? (Remember that 1 cm on your map represents 1 km.)

4. Where do you think is the *worst* place on the island for the spotter to locate? How many kilometers of path would he have to clear from it?

Now consider the problem of where the surfer should build *his* house. He likes the beaches along all three sides of the island and decides to spend an equal amount of time on each beach. To make the paths from his house to each beach as short as possible, he constructs them so that they are ***perpendicular*** to the lines of the beaches.

Perpendicular lines form right angles, 90 degrees (90°) on a protractor or the angle formed by the edges of a file card. An easy way to draw a line that is perpendicular to another line is to use a file card as shown in the figures below. Small squares are usually used to indicate that lines are perpendicular.





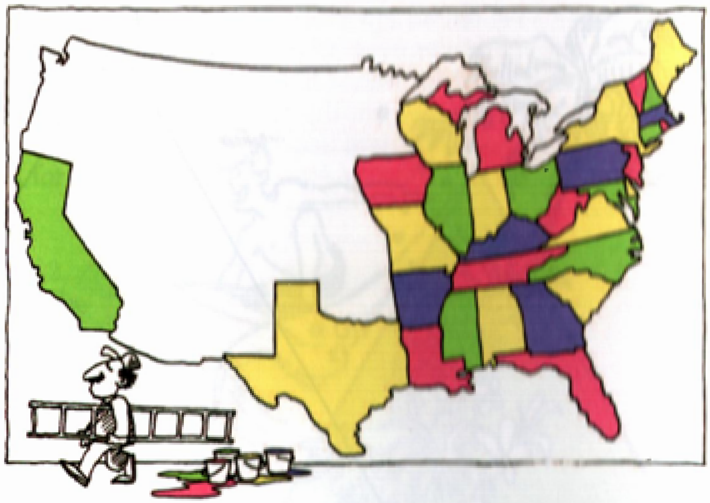
In the figure above, if the surfer built his house at point D, the three paths to the beaches would be as shown. Path DX is perpendicular to beach AC, path DY is perpendicular to beach AB, and path DZ is perpendicular to beach BC. The lengths of the three paths are: DX = 1.4 km, DY = 2.9 km, and DZ = 6.0 km; so their sum is 10.3 km.

The surfer, like the spotter, wants to locate his house so that the sum of the lengths of the paths is a minimum. Where is the best place on the island for him?

5. Use a straightedge and compass to construct another equilateral triangle whose sides are 12 centimeters long. Choose several different points on it; for each point, measure the perpendicular distance from it to each of the sides to the nearest 0.1 cm, and find their sum as illustrated for point D above.

6. On the basis of your work in exercise 5, where do you think is the best place for the surfer to build his house? And how many kilometers of path does he have to clear?

7. Where do you think is the worst place for the surfer to locate? How many kilometers of path would he have to clear from it?



Inductive Reasoning

In thinking about the puzzles of the surfer and the spotter, you first collected some evidence–you made drawings and measured them. Then you looked for conclusions based on what you had observed.

This method of reasoning is used in both science and mathematics. You will be using it throughout this book. The scientist who makes observations, discovers regularities, and formulates general laws of nature calls this process the *scientific method.* Mathematicians refer to it as ***inductive reasoning***. It is the method used for drawing conclusions from a limited set of observations.

We use inductive reasoning regularly in everyday life. Its conclusions are generally reliable and useful, but they are sometimes wrong. For this reason, scientists call such conclusions *conjectures.*

In 1852, a young English mathematician named Francis Guthrie noticed that he could color every map that he could draw or had seen with only four colors in such a way that no two regions having a common border were the same color. The map of the United States of 1852 shown above is an example. Many people tried to draw maps that would require more than four colors, but no one succeeded, which seemed to confirm that "four colors suffice" but did not prove it.

In 1976, more than a century after Guthrie made his conjecture, Wolfgang Haken and Kenneth Appel proved that Guthrie was right. To celebrate Haken and Appel's proof, the University of Illinois used the postage-meter message that you see here. The method that they used, ***deductive reasoning***, uses logic to draw conclusions from statements already accepted as true.

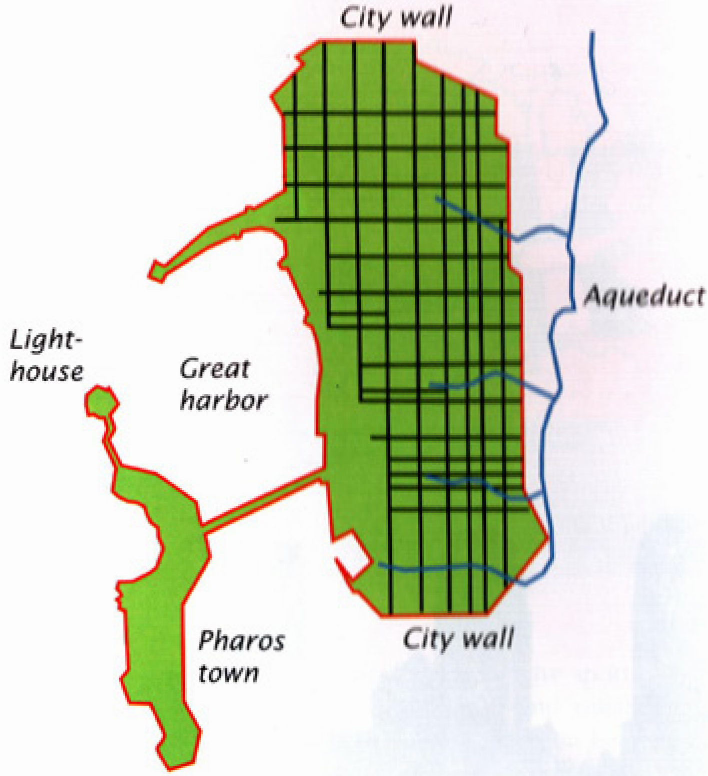
FOUR COLORS
SUFFICE

In addition to using inductive reasoning throughout your study of geometry to discover *what* is true, you will also learn how to reason deductively to be able to understand *why.*

Chapter 1

An Introduction to Geometry

In this chapter, you will see how practical problems ranging from designing a city and measuring the earth to using shadows to tell time led to the development of geometry. The ideas that come from these problems are important because they lead to the solutions of other problems. As you proceed, you will become acquainted with geometric terms and ideas that will be useful as you continue your study of geometry.

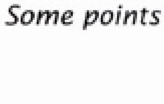


LESSON 1

Lines in Designing a City

Alexandria, the city in which Euclid wrote his famous book on geometry, is named for Alexander the Great. Alexander had conquered much of the ancient world by the time of his death in 323 B.C. He is thought to have planned the streets of Alexandria, which today has become the second largest city in Egypt.

The map above shows the arrangement of the streets in ancient Alexandria. It also suggests some of the basic terms used in geometry. The streets lie along straight *lines;* they intersect in *points* and lie in a common *plane.*



Some points

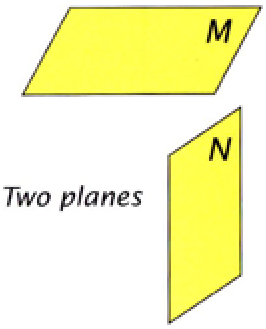
Euclid described a ***point*** as "that which has no part." This description conveys the idea that, when we draw a dot on paper to represent a point, the point, unlike the dot, has a location only, without physical extent. The points in a figure to which we want to refer are labeled with capital letters.

Euclid described a ***line*** as having "breadthless length" and said that "the extremities of a line are points." These statements reveal that he was thinking of what we would now call a *line segment.*

A line segment *A line*

A ***line segment*** is part of a line bounded by two endpoints; it has a length that can be measured. Line segment AB in the figure at the left above has a length of 1 inch. A *line,* however, cannot be measured. The arrows on the figure of the line at the right above indicate that a line extends without end in both directions. These figures also suggest that lines, and hence line segments, are always *straight,* like a stretched string or the edge of a ruler.

Lines are usually named either by two points that they contain, such as line AB above, or by a single small letter. The line in the figure could be simply named line *l*.



Two planes

The map of Alexandria is printed on a flat surface, which illustrates part of a ***plane***. Planes are usually represented by figures such as the two at the right and are usually named, like points, with capital letters. These figures are useful in suggesting that planes are always *flat* but are unfortunately misleading in suggesting that planes have edges. In geometry, we think of a plane as having *no boundaries.* Although the parts of the planes shown here are bounded by edges, the complete planes extend beyond them, just as the line AB extends beyond the endpoints of the segment determined by A and B.

Some additional terms relating *points, lines,* and *planes* are illustrated and defined below.

Definitions

Collinear points *Noncollinear points*

Points are ***collinear*** if there is a line that contains all of them.
Points are ***noncollinear*** if no single line contains them all.

Coplanar points *Concurrent lines*

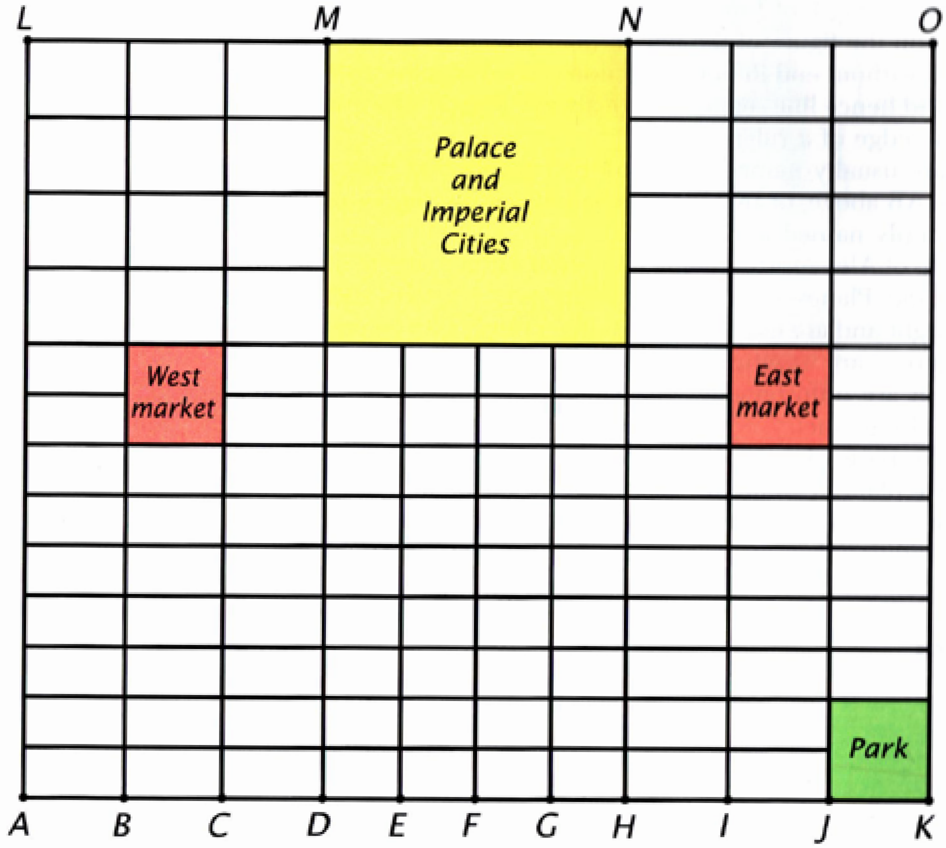
Points are ***coplanar*** if there is a plane that contains all of them.
Lines are ***concurrent*** if they contain the same point.

In the figure at the right above, lines *l, m,* and *n* all contain point P, their common *intersection.*

Exercises

Set I

This map shows the plan of the main streets of the ancient Chinese city Ch'ang-an.



The city of Ch'ang-an

The intersections of the street at the bottom of the map with the streets above it are labeled A through K.

1. What word is used to describe points such as these, given that one line contains them all?

Use your ruler to find the lengths of the following line segments in centimeters.

2. AK.
3. AF.
4. AD.
5. AB.
6. AL.

If the map is accurately drawn and the street from A to K is 6 miles long,

7. how many miles is it from A to F?
8. how many miles does 1 cm of the map represent?
9. how many miles is it from A to L?

10. Is the city square? Why or why not?
11. What regions of the city do appear to be square?

The city, whose name means "long security," was protected by a surrounding wall.

12. Use the lengths of the streets bordering the city to figure out how many miles long the wall was that surrounded it.

If the city were perfectly flat, then all of the points in it would lie in one plane.

13. What word is used to describe points such as these?

14. Are the streets in the map represented by *lines* or *line segments?*

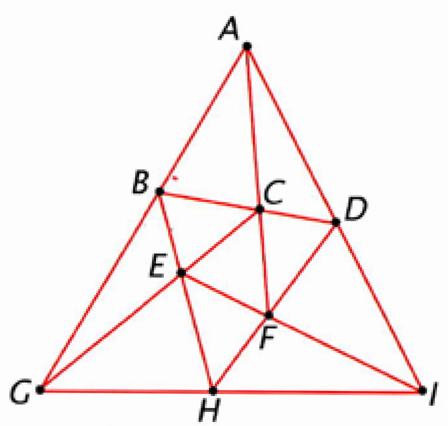
15. What is the difference between a line and a line segment?

Notice that AB, CD, and HK are three different segments, yet they all lie in *one* line.

16. On how many different lines do all the segments in this map lie?

Set II

The points and lines in the figure below are related in some unusual ways.



Each point in which line segments intersect has been labeled with a letter.

17. How many such points does the figure contain?

18. What does the word *collinear* mean?

The figure has been drawn so that each set of points that *appears* to be collinear actually *is.* For example, points E, F, and I are collinear and lie on the line EI.

19. How many sets of three collinear points does the figure contain? List them, beginning with E-F-I.

20. What does the word *concurrent* mean?

How many different *lines* in the figure are concurrent at

21. point A?

22. point B?

23. point C?

24. each of the other points?

And now some very tricky questions about the line segments that can be named by using the letters in the figure.

25. How many different line segments in the figure meet at point A? Name them.

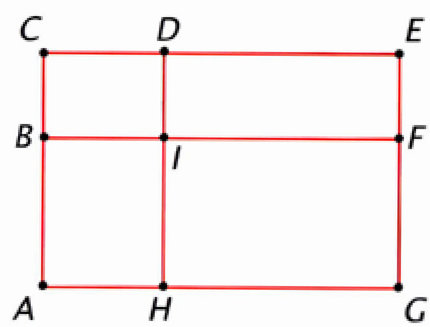
How many different line segments meet at

26. point B? Name them.

27. point C? Name them.

Set III

On a sheet of graph paper, draw a figure similar to, but much larger than, the one below.



1. Use a colored pen or pencil to draw the following three lines:

AI, CF, and DG.

Draw enough of each line so that it crosses the entire figure.

2. What seems to be true about the three lines AI, CF, and DG?

Now use a pen or pencil of a different color to draw these three lines:

AF, CI, and EH.

Extend each line to the edges of the figure.

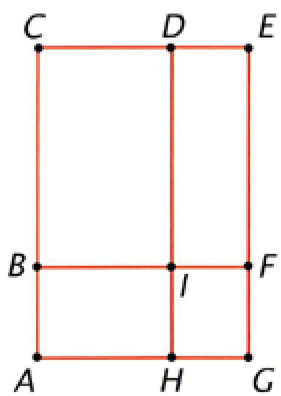
3. Describe what you see.

Now use a pen or pencil of a third color to draw these three lines:

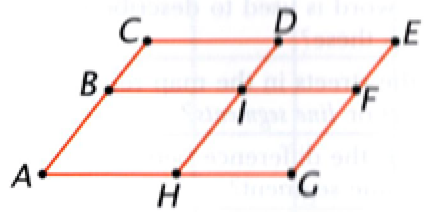
BG, CH, and EI.

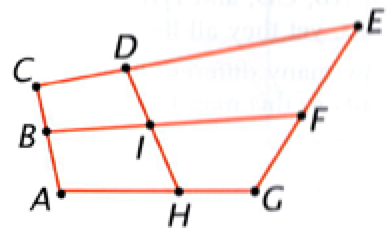
Extend each line to the edges of the figure.

4. What do you notice?
5. Draw the figure again, but change it so that it is taller or shorter, BF is higher or lower, and DH is farther to the left or right. An example of such a figure is shown here, but choose your own.



6. Now try drawing all the lines again and see what happens. What do you think?
7. Try changing the figure in other ways to see what happens to the lines drawn in the three colors. Some examples of altered figures are shown here. (Be sure to make your drawings much larger!)





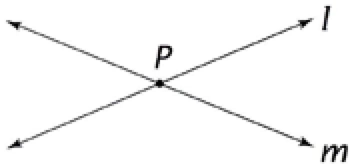
8. What do you think?

LESSON 2

Angles in Measuring the Earth

The ancient Greeks knew that the world was round. Eratosthenes, the head of the library in Alexandria where Euclid lived, figured out a clever way to estimate the distance around the world. His method depended on measuring an *angle.*

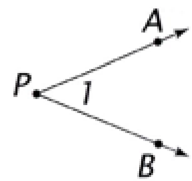
Euclid described an *angle* as "the inclination to one another of two lines in a plane which meet one another." If we draw a figure showing two lines, *l* and *m,* meeting at a point P, several angles are formed.



A ray

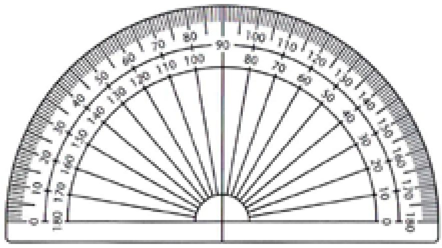
The figure can also be described as consisting of four *rays* starting from point P. As the figure suggests, a ***ray*** is part of a line that extends endlessly in one direction. To refer to a ray, we always name its endpoint first, followed by the name of any other point on it. The name of the ray in the first figure at the right is PA.

An ***angle*** is a pair of rays that have the same endpoint. The rays are called the *sides* of the angle and their common endpoint is called the *vertex* of the angle. Angles can be named in several ways. Using the symbol ∠ to mean "angle," we can name the angle in the second figure at the right ∠P or ∠1 or ∠APB or ∠BPA. Notice that, if the angle is named with three letters, the vertex is named in the middle.

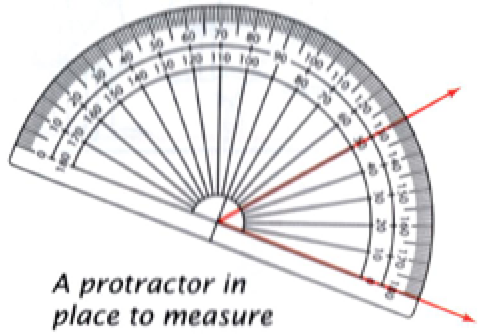


Two rays that form an angle

Before learning how Eratosthenes measured the earth, we will review the way in which angles are measured. To measure an angle, we need a *protractor.*

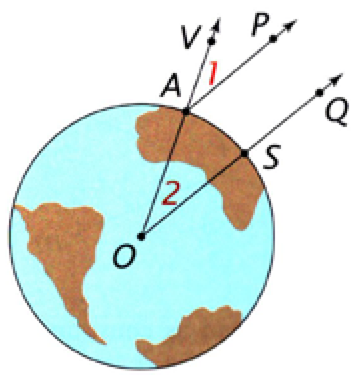


A protractor



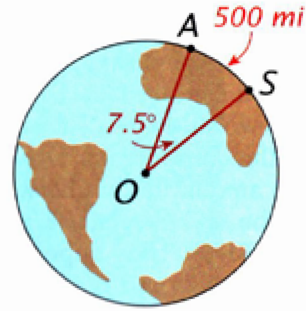
A protractor in place to measure an angle

The protractor measures angles in *degrees* and has two scales, each numbered from 0 to 180. The center of the protractor is placed on the vertex of the angle so that the 0 on one of the scales falls on one side of the angle. The number that falls on the other side of the *same* scale gives the measurement of the angle in degrees. Look carefully at the scales on the protractor in the figure above to see why the angle has a measure of 50° and *not* 130°.



The diagram above illustrates the method that Eratosthenes used to measure the earth. He knew that Alexandria was about 500 miles north of the city of Syene (now called Aswan). The points A and S represent Alexandria and Syene, respectively, and the rays AP and SQ represent the direction of the sun as seen from each city. At noon on a certain day of the year, the sun was directly overhead in Syene, as shown by SQ. In Alexandria at the same time, the direction of the sun was along AP, in contrast with the overhead direction AV.

Eratosthenes measured the angle between these two directions, ∠1, and found that it was about 7.5°. Euclid had shown why it was reasonable to conclude that ∠1 was equal to ∠2, the angle with its vertex at the center of the earth. By dividing 7.5° into 360° (the measure of the degrees of a full circle), we get 48; so the full circle is 48 times the angle from Alexandria to Syene:



$$48 \times 7.5° = 360°.$$

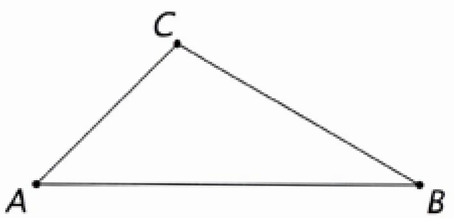
Eratosthenes concluded that the earth's circumference (the entire distance around the circle) was 48 times the distance from Alexandria to Syene:

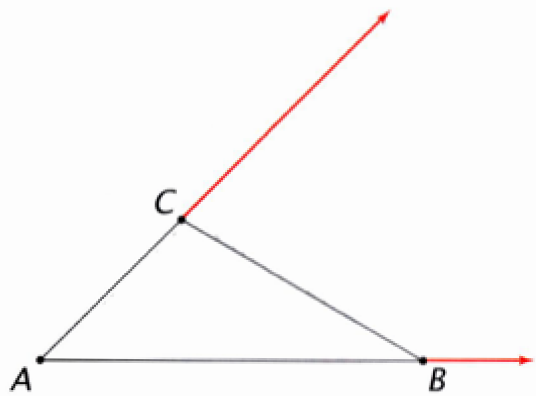
$$48 \times 500 \text{ miles} = 24{,}000 \text{ miles}.$$

The modern value of the circumference of the earth is about 25,000 miles; so Eratosthenes' estimate was remarkably accurate.

Exercises

Set I





Triangle Measurements. Each side and angle of the triangle above has a different measure. If you simply look at the figure without doing any measuring,

1. which side is the longest?
2. which angle is the largest?
3. which side is the shortest?
4. which angle is the smallest?
5. Use your ruler to measure the sides of the triangle, each to the nearest 0.1 cm.

Most protractors are too large to be able to measure the angles of a figure such as this triangle easily. For example, to measure ∠A, it is easiest to extend its sides as shown in color in the figure at the right above.

6. Name the two rays that are the sides of ∠A.
7. Use your protractor to measure ∠A.
8. Carefully trace triangle ABC on your paper and then extend the sides of ∠B.
9. Name the two rays that are the sides of ∠B.
10. Measure ∠B.

Go back to the figure you drew for exercise 8 and extend the sides of ∠C.

11. Name the two rays that are the sides of ∠C.
12. Measure ∠C.

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HAROLD R. JACOBS is teacher of mathematics and science, writer, and well-respected speaker. He received his B.A. from U.C.L.A. and his M.A.L.S from Wesleyan University. His other publications include *Mathematics: A Human Endeavor*, *Elementary Algebra* and articles for *The Mathematics Teacher* and the *Encyclopedia Britannica*. Mr. Jacobs has received the Most Outstanding High School Mathematics Teacher in Los Angeles award, the Presidential Award for Excellence in Mathematics Teaching, and was featured in the book *101 Careers In Mathematics* published by the Mathematical Association of America.

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